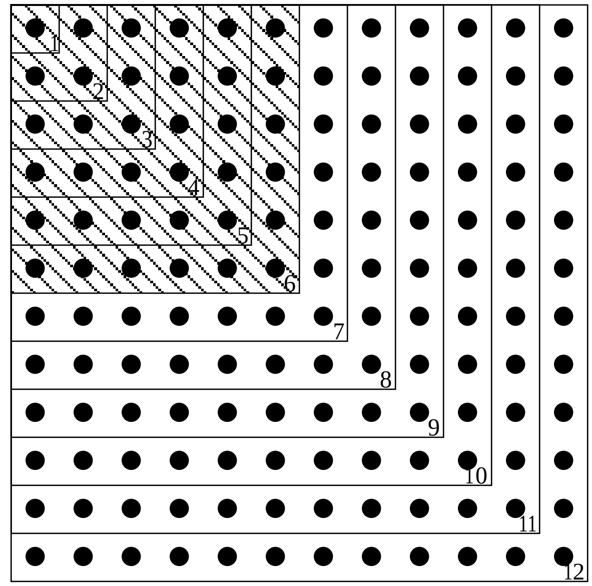
Squared Squares Worksheet

Note to teachers – You have permission to copy this worksheet for use in your own classroom only.

“Squared Squares” is based on the square numbers, those that can be expressed as the product of an integer with itself. In the puzzle, a product of two numbers is represented by an area. For example, 6 x 6 is represented by the shaded area at right. In this diagram, a dot has been centered on each unit area, so the area can be obtained by counting the dots.

Count the number of dots in each L-shaped region of the figure and write these numbers in the blank spaces in the table below, along with the square of each number. What do you notice about the numbers you counted?



Integer	1	2	3	4	5	6	7	8	9	10	11	12
Square	1	4	9									
Dots in “L”	1	3	5									

The diagram above shows that a square number can be expressed as the sum of consecutive odd integers. For example, $5^2 = 1 + 3 + 5 + 7 + 9$. Try to write a simple expression in terms of k to replace the “?” in the formula below, expressing this sum in a general form for any square n^2.

$$n^2 = \sum_{k=1}^{n} ?$$

The “four-square theorem”, proven by Joseph Louis Lagrange in 1770, states that every positive integer can be expressed as the sum of at most four square numbers. Using the squares of the integers 1 through 12, find these sums of squares for each positive integer below. The first one is done as an example. Remember that the maximum number of allowed squares is four.

$19 = 16 + 1 + 1 + 1 = 4^2 + 1^2 + 1^2 + 1^2$

80 =

44 =

150 =

231 =

142 =